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Updating Occam's razor:
Computational complexity, causal processes and the laws of nature

It is often claimed that Occam's razor should govern how we pick up the best mechanistic model, set of laws, or causal explanation. Hence complexity is a principle for theoretical choices: whatever „complexity" means, reducing complexity should guide our epistemic decisions. Especially, between several hypothesized causal processes or causal laws that are equally empirically predictive, this principle tells us to chose the simplest. But of course there are various conceptions of complexity − hence of simplicity − and they support various versions of Occam's razor − which therefore may eventually prove underdetermined. In this talk, we question the value of Occam's razor if it is applied as a simple epistemic rule for causal modeling, causal inference and theory choice. We will argue that, even before examining the proper concept of complexity requested to implement the principle, one should question the object on which measures of complexity are applied − especially, whether it concerns the hypothesized causal laws or processes, or data, or both. From this viewpoint, it will appear that whatever concept and measure of complexity one has, requisites about some trade-of between distinct complexities should govern the theory or model choice. Unlike simple versions of Occam's razor such a modified principle is not categorical but context dependent.

We show first that, from a purely computational viewpoint, assessing the complexity of a process or a rule actually involves two sorts of complexity, whose relationship is governed by specific constraints. We then present a situation in evolutionary biology that displays such a formal property, before drawing some general epistemological consequences based on these case studies.

1. Cellular automata (CA) are the paradigmatic example of a connectionist model of computation, which are believed to bring out complex, emergent features through the interaction of a great number of simple units, the cells, according to simple rules. The simplest CA that still show complex behavior are the elementary cellular automata (ECA). ECA consist of a one-dimensional array of cells, which can be in either of two states, usually denoted by 0 and 1, and which interact with themselves and their next-nearest neighbors. Obviously ECA can be used as computational devices: some input is given to them through their initial configurations, then processed according to the interaction rules and finally, when a specific configuration has been reached, the output of this computation is read out.

How complex is this computational process? According to standard computability theory, there is a hierarchy of automata, corresponding to the Chomsky hierarchy of classes of formal grammars, starting from finite state automata up to the Turing machine, which, according to the Church-Turing thesis, produces the most complex behavior possible by a computational process. It has been conjectured by Wolfram [1] and proven by Cook [2] that a certain ECA- rule, called rule 110, is computationally universal in the Turing machine sense. The proof proceeds by reducing the system to another computational model, the \cyclic tag system", already known to be universal. In order to do so, there must be a translation from one system to the other, especially the input and the output of the ECA must be encoded and decoded. This encoding/decoding process is itself computational. Then, in order to speak of a „complex" process − namely, the universality of ECA rule 110 - the encoding/decoding process must be of a lower complexity than the process actually investigated. Here, this means that the encoding /decoding process, i.e. the process that achieves the translation of one computational system into the other, must be executable by an automaton that is lower in the automata hierarchy. Otherwise, an utterly trivial process could become very complex, namely universal, as all the computational complexity already lies in the process achieving the encoding/decoding of the computational system. In short, if we want to quantify the complexity of the computational process, we have to set this complexity in relation to the complexity of the process that codes for the initial input or initial condition.

2. In evolutionary biology, the Modern Synthesis theory uses population genetics models (Fisher 1930) that causally explain evolution by modeling the change in allelic frequencies in a population (at one or two loci). Adaptive change of phenotypic traits in various species, as

well as phenomena such as speciation, polymorphism, etc., are thus explainable by applying equations proper to various population or quantitative genetics models such as the Fisher-Wright model, the Moran model, etc. (they include Markov chain models, diffusion equations, etc.); these are simpler than a model that would integrate all differential equations describing all individual trajectories and interactions in the population. . . Hence mechanistic models of change at the genotypic level explain phenotypic changes. Even though the scope of the Modern Synthesis is currently discussed, this has been explanatory and predictive for decades. However, it progressively appeared that the validity of these models relies on assumptions about the relation between genotypes and phenotypes, as well as some genomic properties: if the genotype space is too rugged (meaning that alleles are too much depending on other alleles' effect for their fitness value) [3], or if the genotype-phenotype map is too complex [4], then the Modern Synthesis simplifications don't apply and the simple models of population genetics can't explain evolution any more. Here too, trade-offs between the hypothesized process and the complexity of the boundary or initial conditions take place that implicitly constrain the model choice. Hence, it appears that if we take the complexity of a process as a causal chain of events that fall under a causal law, the complexity of such law must be set in relation to the complexity of the initial or boundary conditions or the complexity of the processes generating them. Naturally, this complexity should be describable in the same terms. Then, Occam's razor should be by taking into account the trade-o between the complexities of hypothesized causal laws/rules/processes and boundary/initial conditions. What is to be minimized is the complexity of a combination of these processes, not the complexity of a singular process, no matter how complexity is exactly denied. What then governs theory choice is an optimal trade-o rather than an unconditional minimization of complexity.

References:

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